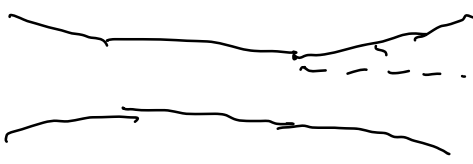


# Lasers

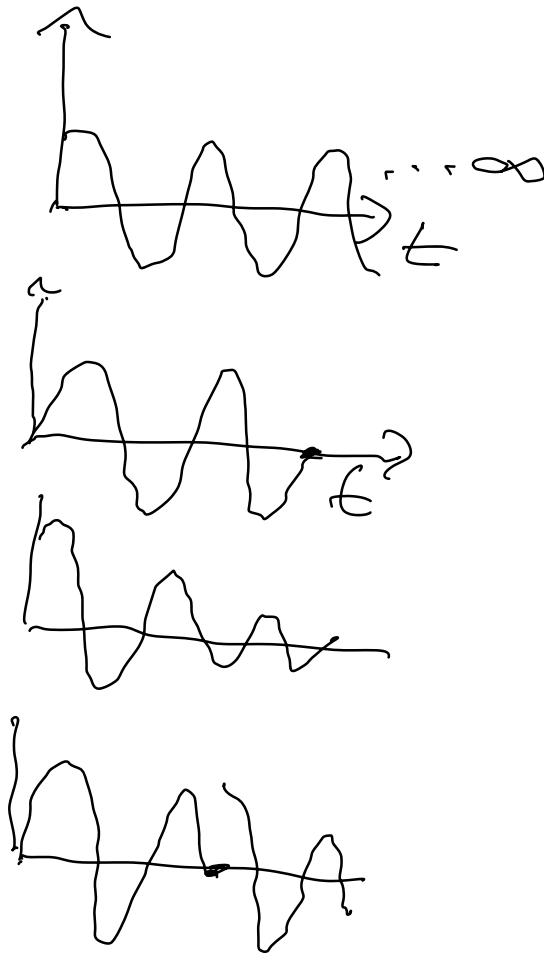
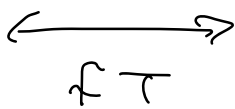
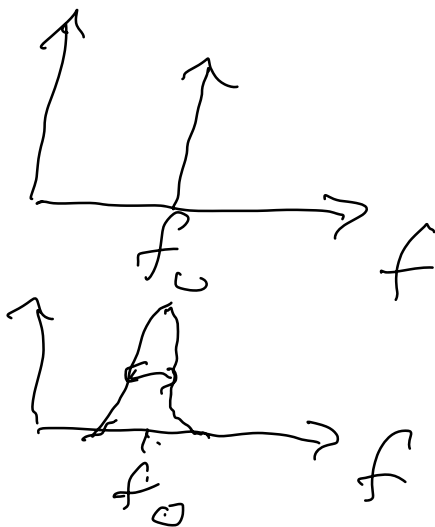
coherent  
monochromatic  
collimated -

directional



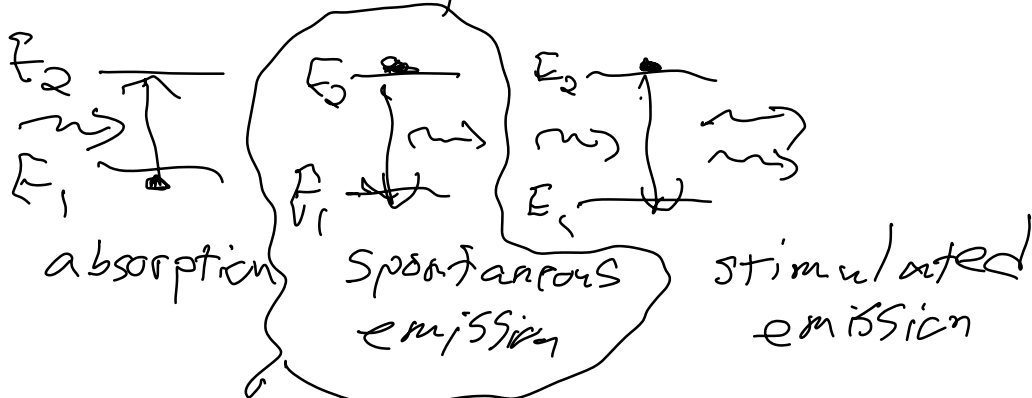
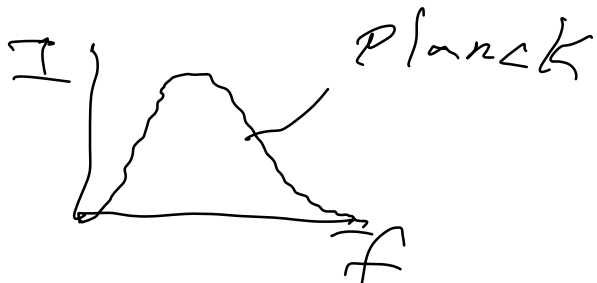
diffraction  
 $\theta \approx \frac{\lambda}{D}$

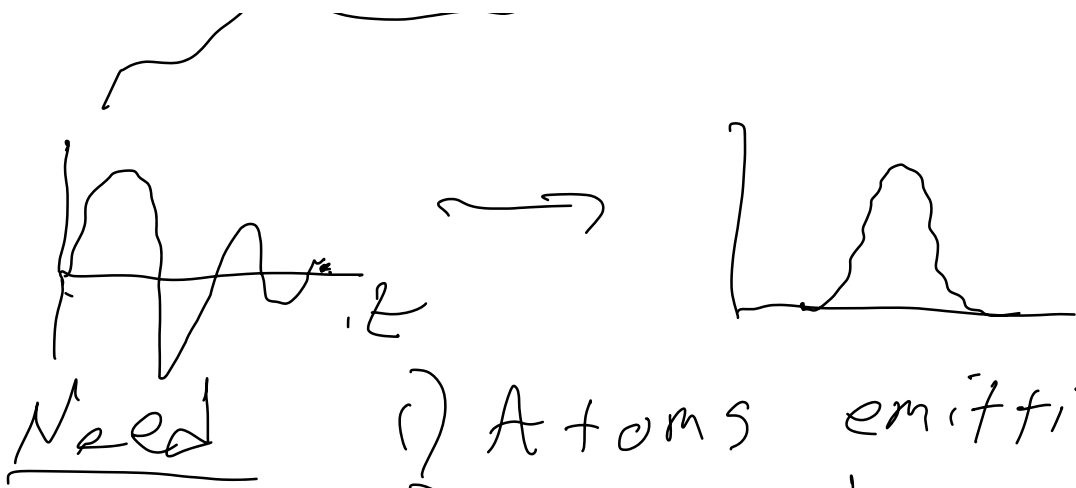
$$c = f \lambda$$



$$\Delta \omega \Delta t \geq \frac{1}{2}$$

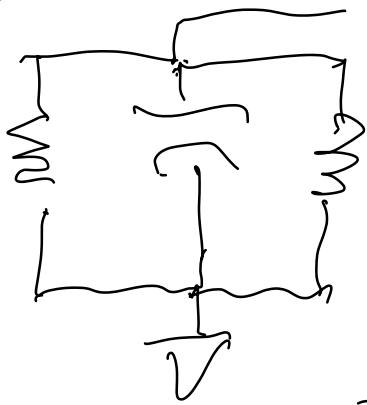
$$\omega = 2\pi f$$





- Need
- 1) Atoms emitting in phase
  - 2) Sustained emission

~ Resonator



$$v_L = L \frac{di}{dt}$$

$$i_C = C \frac{dv}{dt}$$

$$\frac{v}{R} + C \frac{dv}{dt} + \int \frac{v_L}{L} dt, i=0$$

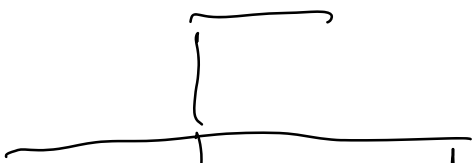
$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

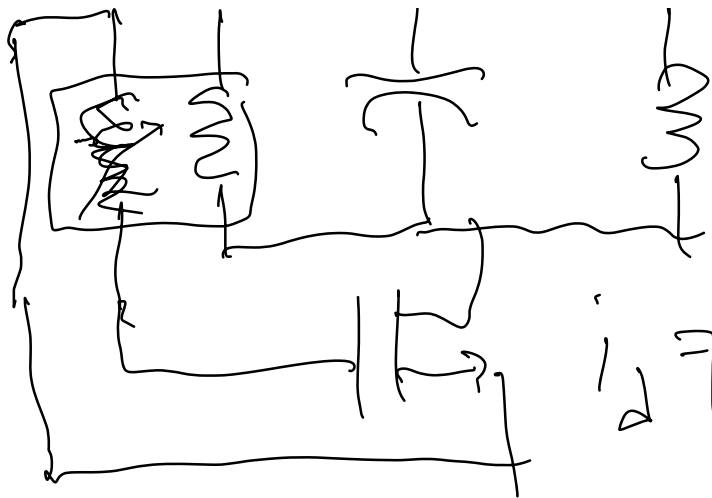
$$s_{1,2} = -\alpha \pm j\sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{L}{2RC}$$

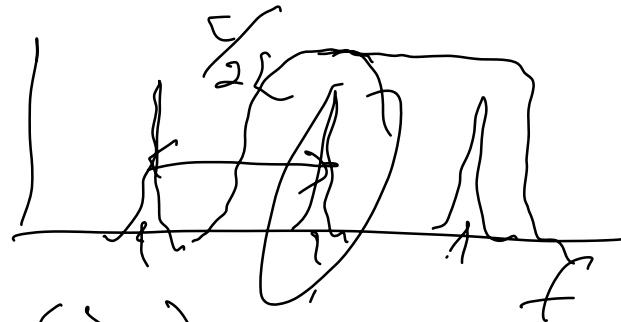
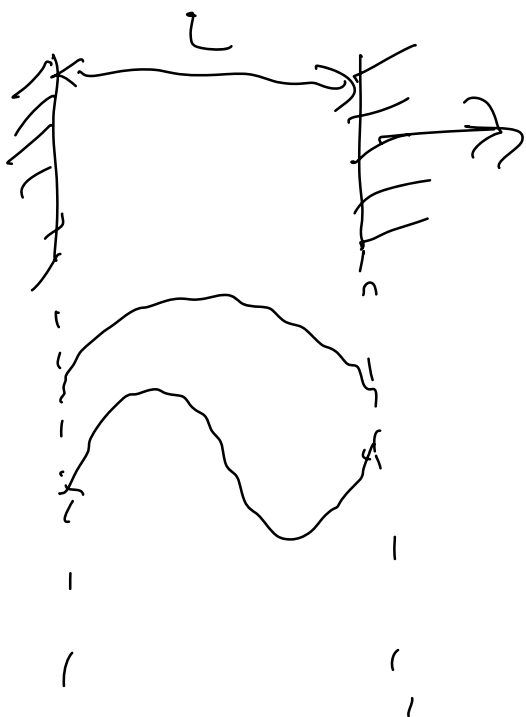
$$\omega_0 = \frac{1}{\sqrt{2C}}$$

$$s = j\omega_0$$





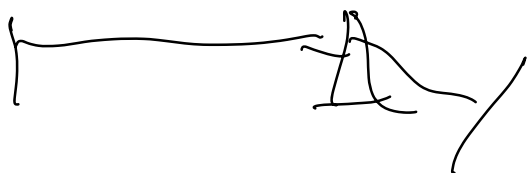
$$i_D = g_m V_{gs}$$



$$n \left( \frac{\lambda}{2} \right) = L$$

$$c = f \lambda$$

$$f_n = n \frac{c}{2L}$$



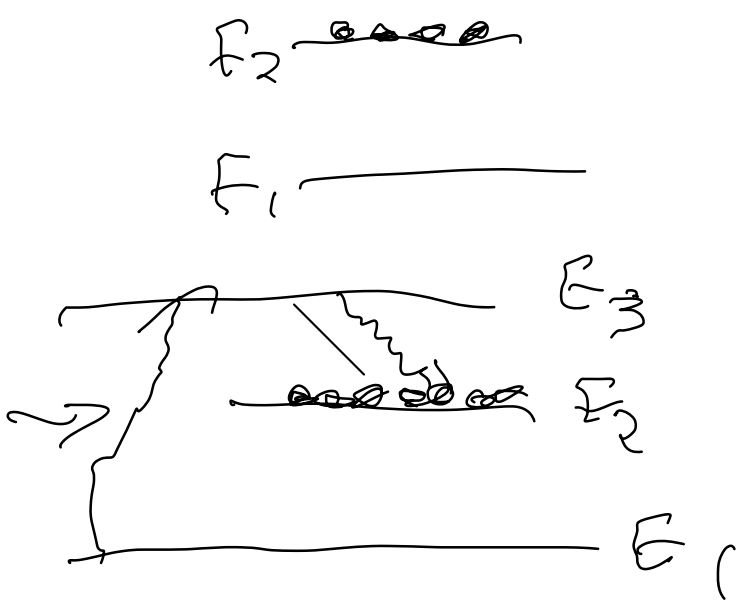
Einstein coefficient

$$B_{12} = B_{21}$$

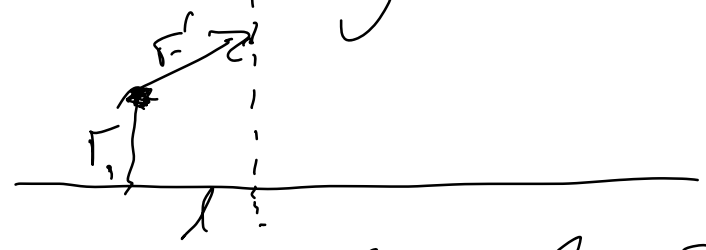
absorption      Spont.  
emission

population inversion

$\geq 3$  levels



Ray tracing



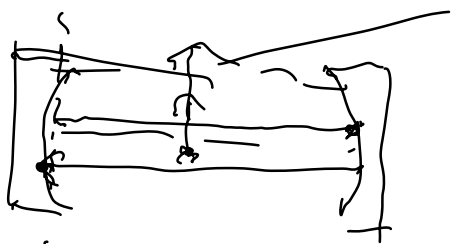
$$\begin{bmatrix} r_1 \\ r_1' \end{bmatrix} \rightarrow \begin{bmatrix} r_1 + \rho r_1' \\ r_1' \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$$

$$[ ] = [ ] [ ] [ ]$$

$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \text{ thin lens}$$

$$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix} \text{ mirror}$$



$$\begin{bmatrix} 1 & 0 \\ -2/R_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{RT}$$

$$\begin{aligned} r_{s+1} &= A r_s + B r'_s \\ r'_{s+1} &= C r_s + D r'_s \end{aligned}$$

$$\Gamma_{s+2} = (A+D)\Gamma_{s+1} - \Gamma_s$$

$$\Gamma_{s+2} - (A+D)\Gamma_{s+1} + \Gamma_s = 0$$

$$\Gamma_s = \lambda^s$$

$$\lambda = \frac{A+D}{2} \pm \sqrt{\frac{(A+D)^2}{4} - 1}$$

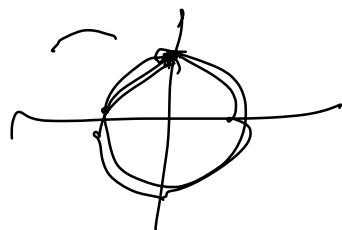
$$m = \frac{A+D}{2}$$

$$\lambda = m \pm \sqrt{m^2 - 1}$$

$$m = 0$$

$$\lambda = j = e^{j\frac{\pi}{2}}$$

$$\Gamma_s = \lambda^s = e^{j\frac{\pi}{2}s}$$



$$m > 1$$

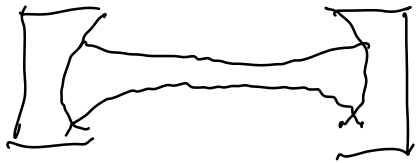
$$\Gamma_s = \lambda^{s+1}$$

$$\boxed{|m| < 1}$$

$$\boxed{\left| \frac{A+D}{2} \right| < 1}$$

Stability  
Condition

real Gaussian beams



$$E(x, y, z) = E_0 \psi(x, y, z) e^{-jkz}$$

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 \psi - j2k \frac{d\psi}{dz} = 0$$

$$\psi = \exp(-jP(z)) \exp\left(-j \frac{k r^2}{2g(z)}\right)$$

$$g(z) = z + jz_0$$

$$z_0 = \frac{\pi W_0^2}{\lambda}$$

Rayleigh range  
confocal parameter

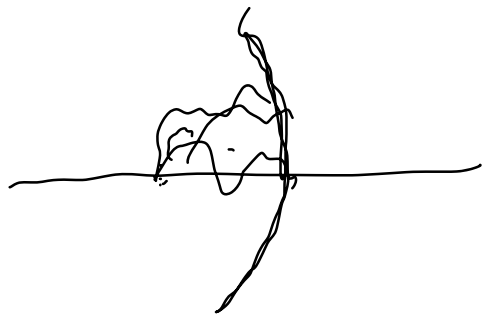


$$w_0^2 = \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]$$

$$R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]$$

$$\frac{1}{g(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi w^2(z)}$$

$$E(r, t) = \frac{w_0}{w} \underbrace{e^{-\frac{r^2}{w^2(z)}}}_{\text{Gaussian shape}} \times$$



$$e^{-j \frac{k r^2}{2 R(z)}} \quad \text{curvature field}$$

$$e^{-j \tan^{-1} \frac{z}{z_0}} \quad \text{extra phase}$$

1) derived

2) ABCD

$$g_2 = \frac{A g_1 + B}{C g_1 + D}$$

$$\frac{1}{g_2} = \frac{C + \frac{v}{g_1}}{A + \frac{B}{g_1}}$$